## Maximum likelihood estimation

Method of Moments

Let be a where is a vector of parameters.

The likelihood function:

The log-likelihood function

The maximum likelihood estimator of should satisfy:

Thus

Now is usual called the score vector. That is,

And so the MLE for can be found by equating the score to zero. That is,

The variance of an ML estimator, , is calculated by the inverse of the Information matrix:

The Information matrix is the negative of the expected value of the Hessian matrix:

where Hessian is the matrix of second derivatives of the likelihood with respect to the parameters:

Thus, the variance-covariance matrix of is:

Example. **The Bernoulli distribution**.

Consider a random sample of size from the distribution. The probability density function here is:

Our likelihood and log-likelihood functions are:

and

=

The score function is:

That is,

Equating this to zero, we find

What is the variance of ?

The variance of the MLE is

Example. **The Poisson distribution.**

Consider a random sample of size from the distribution. The probability density function here is:

Our likelihood and log-likelihood functions are:

and

The score function is:

That is,

Equating this to zero, we find

=0

What is the variance of

The standard error of an estimator

The standard error of

A confidence interval for

For the Poisson distribution the 95% CI for is

Example. The exponential distribution

The random sample

The Likelihood:

The log-likelihood

The score function

The Hessian

The Information

The variance of :

The standard error of :

The 95% CI for

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Example. The Cauchy

The random sample

The Likelihood:

The log-Likelihood:

The score function :

Find the Score and Hessian for the Cauchy distribution. By Thursday this week